On Centres of Aggregation and Dissociation.

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I. *Æthereal Influences in the Solar System.*

The velocity of rotation varies as the square root of the velocity of gravitating fall. Therefore, if the velocity of planetary revolution ($\sqrt{gr}$) at Sun's equatorial surface be taken to represent the velocity of æthereal rotation at the same point, the present æthereal atmospheric limit, at which the equatorial velocity of rotation would be equal to that of the æthereal undulations which drive particles towards centres of inertia, is near the outer limit of the asteroidal belt, at ($\sqrt{\text{Light-Modulus}}$ = 688.3 solar radii = 3.204 Earth's mean vector-radii). The mean proportional between Earth's mean distance and Saturn's* secular aphelion is 3.216........(1.)

We have found that the velocity at the æthereal atmospheric limit (the velocity of light), is the limiting mean radial velocity, at the point of equilibrium between the velocity of complete dissociation and the velocity of incipient aggregation. The limiting tangential velocity, at the point of equilibrium between complete aggregation and incipient dissociation, is, therefore,† $\frac{1}{\pi}$ of the velocity of light, which is the velocity of æthereal rotation at 219.09 solar radii, Earth's mean distance being 214.86........(2.)

We have also found‡ that Jupiter, the largest extra-asteroidal, and Earth, the largest intra-asteroidal planet, are connected by a common limiting radial velocity, the indications pointing to nucleal or rotating influences in the case of Earth, and to atmospheric or orbital influences in the case of Jupiter. Circular-orbital velocity varies as $g^{\frac{1}{4}}$. The relations of g to $\sqrt{M}$ (1) and of Earth's position to the unit of solar aggregation (2), lend importance to the approximate equality between log. $(\sqrt{M})^{\frac{1}{4}}$ = .709444 and log. rad. vec. (♃ : ⊕) = .716237..(3.)

Alexander showed§ that Jupiter and Saturn are so related as to suggest a possible early mutual participation in a common nucleal *vis viva;* as if they had been formed, by interior and exterior condensation, from the same nebulous belt. I have shown‖ that the atmospheric radius varies as the $\frac{4}{3}$ power of the nucleal radius. Therefore, if $M^{\frac{1}{8}}$ represents Jupiter's posi-

*"Fundamental Propositions of Central Force," (*ante.* p. 298-310) VI.

† Ib., V.

‡ Ib., V—VII, Illustrations.

§ Smithsonian Contrib., 280, p. 38.

‖ "Fundamental Propositions," X.

tion (3) at the extremity of a nucleal radius, the corresponding atmospheric radius $= (\sqrt[8]{M})^{\frac{4}{3}} = M^{\frac{1}{6}}$, of which the logarithm is .945926. The log. of ♄ secular perihelion ÷ ⊕ mean radius vector is .941236............(4.)

The secular perihelion of Venus is near the æthereal nucleal limit (1, 4). Log. $(\sqrt{M})^{\frac{3}{4}} = 2.128332$, log. ♀ sec. perihelion being 2.159680. 2.159680 — 2.128332 = .031348 = log. 1.07483. ♀ secular aphelion ÷ mean r. vec. = 1.07633..(5.)

These approximations point to æthereal influences on the principal planets. both in the supra- and in the infra-asteroidal belts, and to early special nucleal condensation in the inner belt. The latter indication is strengthened by the greater density of the interior planets, by the many harmonic relations which are based on Earth's distance as a primitive unit, and by Earth's position near the centre of the infra-asteroidal belt. Mercury's secular perihelion (.29743) + Mars's secular aphelion (1.73633) = 2.03376 ; 2.03376 ÷ 2 = 1.01688. Earth's present aphelion = 1.01678...............(6.)

The nucleal-atmospheric relations of Earth and Jupiter (3) are further shown by the fact that a nucleal expansion of Sun to Earth's secular perihelion (200.307 solar radii) would involve an atmospheric expansion to ($200.307^{\frac{4}{3}} = 1172$). Jupiter's mean aphelion = 1166.61 solar radii....(7.)

The present Light-Modulus (log. M ÷ $\odot r$ = 5.675554) : Earth's accelerative radius (log. $2\rho^2 = 4.965340$) : : Jupiter's secular aphelion (log. = .741881) : Earth's secular aphelion (log. = .028463)................(8.)

Earth's rotating, relatively to its orbital velocity, has been accelerated 366.256 times, since its theoretical nebular rupture. This represents the ratio of Earth's nucleal-rupturing to Sun's aggregating radius (2). For if we let ρ = superficial radius and velocity of perfect fluidity in the æthereal nucleus (1), $\rho\sqrt{2}$ = radius of dissociating velocity, and $2\rho^2$ = radius of rupturing *vis viva*. Log. (ρ = ⊕ secular perihelion) = 2.301695. Log. ($2\rho^2$ ÷ 219.0894) = 2.563791 = log. 366.253.......................(9.)

The increased acceleration of Jupiter's angular velocity, relatively to its nucleal companion Earth, is such as would be due to the difference of orbital velocities at the outer and inner edges of the Jovi-Telluric belt. Log. ♃ secular aphelion (.741881) — log. ⊕ sec. perihelion ($\bar{1}.969540$) = log. 2.433^2. 24 hours ÷ 2.433 = 9 h. 51 m. 49 s.......................(10.)

Jupiter's rotating, relatively to its orbital velocity, has been accelerated in the ratio of its mean rupturing radius to Sun's aggregating radius. For log. (ρ = ♃ mean perihelion) = 3.029231 ; log. ($2\rho^2$ ÷ 219.0894) = 4.018865 = log. 10443.97 ; 4332.585 dy. ÷ 10443.97 = 9 h. 57 m. 22 s. See (7) (35) (48)..(11.)

Saturn's rotating, relatively to Jupiter's orbital velocity, has been accelerated in the ratio of Jupiter's nucleal rupturing to Sun's aggregating radius. For log. (= ♃ sec. per.) = 3.021137 ; log. ($2\rho^2$ ÷ 219.0894) = 4.022677 = log. 10061.83 ; 4332.585 dy. ÷ 10061.83 = 10 h. 20 m. 3 s........(12.)

Saturn's rotating, relatively to its orbital velocity, has been accelerated

in the ratio of its initial-rupturing radius to Earth's radius of rupture. For log. ($\rho =$ ♄ sec. aph.) $= 3.346812$; log. $(2\rho^2 \div 200.307) = 4.391929 =$ log. 24656.36 ; 10759.22 dy. $\div$ 24656.36 $=$ 10 h. 14 m. 4 s ....(13.)

The rotating velocity of Mars, relatively to its orbital velocity, has been accelerated nearly in the ratio of its nucleal-rupturing radius to Earth's secular aphelion. For log. ($\rho =$ ♂ sec. per.) $= 2.449775$; log. $(2\rho^2 \div 229.413) = 2.839962 =$ log. 691.77 ; 686.98 dy. $\div$ 691.77 $=$ 23 h. 49 m. 49 s...(14.)

The rotating velocity of Venus, relatively to its orbital velocity, has been accelerated in the ratio of its mean rupturing radius to Earth's mean perihelion. For log. $\rho = 2.191493$; log. $(2\ {}^2 \div 207.583) = 2.366824 =$ log. 232.715 ; 224.7 dy. $\div$ 232.715 $=$ 23 h. 13 m. 36 s..................(15.)

The rotating velocity of Mercury, relatively to its orbital velocity, has been accelerated in the ratio of its initial-rupturing radius to Sun's aggregating radius. For log. ($\rho =$ ☿ sec. aph.) $= 1.990608$; log. $(2\rho^2 \div 219.0894) = 1.941619 =$ log. 87.422 ; 87.97 dy. $\div$ 87.422 $=$ 24 h. 9 m. 2 s...(16.)

Jupiter's secular aphelion (5.5193) is a mean proportional between Earth's mean distance, and Neptune's secular aphelion (30.4696). See also, *infra* (27) to (29)..(17.)

The secular perihelion of Uranus, or its locus of nebular rupture (17.688), is at the centre of the supra-asteroidal belt. For Neptune's secular aphelion (30.470) $+$ Jupiter's secular perihelion (4.886) $= 35.356$; $35.356 \div 2 = 17.678$..(18.)

The secular perihelion of Uranus is also a mean proportional between Saturn's secular aphelion (10.343) and Neptune's mean aphelion (30.336)..(19.)

The centres of the outer and inner planetary belts are so related that the mean distances of Uranus (19.184) and Earth's secular perihelion (.932), are at apses of a major-axis which would be traversed by light undulations in the time of planetary revolution at Sun's surface. $(19.184 + .932 = 20.116$; $688.3 \times 2\pi \div 214.86 = 20.128)$ ....(20.)

The major-axis of the November meteoric orbit is also nearly equivalent to the major-axis of these primeval light undulations. For the meteoric period $= 33.25$ yr.; $2 \times 33.25^{\frac{2}{3}} = 20.68$..........................(21.)

When Sun's surface of dissociation was at the extremity of Earth's mean radius vector, the locus of complete association, or the vertex of the stellar-solar paraboloid*, was at Mercury's present perihelion (.3187). $1 \div \pi = .3184$..(22.)

The orbital velocity varies (3) as the one-fourth power of the gravitating velocity. The orbital velocity at the mean aphelion of the intra-asteroidal belt, is equivalent to the mean velocity of the centripetal gravitating im-

* Ib. X, Illustrations. By an inadvertence it was stated that there are nine abscissas between Neptune and α Centauri. There were nine in my original paraboloid (Proc. S. P. A., Sept.20, 1872), but if the vertex is taken at the locus of complete solar aggregation there are eighteen.

pulses beyond the belt. For log. (sec. aph. ♆ × sec. aph. ♂)$^{\frac{1}{8}}$ = .215437; log. mean aph. ♂ = .215944.......................................(23.)

The mean velocity of the centripetal gravitating impulses in the principal nucleal belt is equivalent to the same orbital velocity. For log. (sec. aph. ♄ × mean ♃)$^{\frac{1}{8}}$ = .216362.......................................(24.)

There is, therefore, an equivalence between the mean exterior and the mean nucleal gravitating impulses, beyond the Telluric belt. For log. (sec. per. ♆ × sec. aph. ♂)$^{\frac{1}{2}}$ = .855866; log. (sec. aph. ♄ × mean per. ♃)$^{\frac{1}{2}}$ = .855450.......................................(25.)

The orbital velocity varies as the one-half power of the rotating velocity. The mean orbital velocity due to nebular action in the Neptuno-Uranian belt, is equivalent to the rotating velocity at the locus of nebular rupture in the principal nucleal belt. For log. (mean per. ♆ × mean ♅)$^{\frac{1}{4}}$ = .689039; log. sec. per. ♃ = .688982.......................................(26.)

The initial rupturing position of the centre of planetary mass (17) is determined by the mean influence of the intra-asteroidal centres (6), the supra-asteroidal centre (18), and the nebular centre of planetary inertia (♄). For log. (mean ⊕ × sec. per. ♅ × mean ♄)$^{\frac{1}{3}}$ = .742338; log. sec. per. ♃ = .741881.......................................(27.)

The same position is also a mean proportional between the centre of the supra-asteroidal and the outer limit of the intra-asteroidal belt. For log. (sec. per. ♅ × sec. aph. ♂)$^{\frac{1}{2}}$ = .743575.......................................(28.)

The nebula-rupturing position of the centre of planetary mass is at the centre of the initial planetary system. For sec. aph. ♆ (30.470) — sec. aph. ♅ (20.679) = 2 × sec. per. ♃ (4.886).......................................(29.)

The initial position of mean planetary inertia is determined by the mean positions of the rupturing loci of the two principal two-planet belts. For log. (♅ × ♃)$^{\frac{1}{2}}$ = .999583; log. mean aph. ♄ = 1.000003.......................................(30.)

The atmospheric limit (4) of the infra-asteroidal belt, is determined by positions of Sun, Jupiter, and Neptune. For log. ((♃ × ♆)$^{\frac{1}{2}}$ ÷ ☉ r) = 3.429079; log. (sec. aph. ♂ ÷ ☉ r)$^{\frac{4}{3}}$ = 3.429048.......................................(31.)

The atmospheric limit of the initial position of the infra-asteroidal centre, is determined by positions of Sun, Jupiter, and Saturn. For log. ((sec. per. ♃ × sec. per. ♄)$^{\frac{1}{2}}$ ÷ ☉ r) = 3.147264; log. (sec. aph. ⊕ ÷ ☉ r)$^{\frac{4}{3}}$ = 3.147491.......................................(32.)

The atmospheric limit of the initial tendency to infra-asteroidal rupture, is determined by positions of Sun, Jupiter, and Earth. For log. ((mean per. ♃ × ⊕)$^{\frac{1}{2}}$ ÷ ☉r) = 2.680693; log. (sec. aph. ☿ ÷ ☉r)$^{\frac{4}{3}}$ = 2.680615.......................................(33.)

The atmospheric limit at the inner locus of infra-asteroidal rupture, is

the nucleal rupturing limit, relatively to Earth, of Mars. For log. (sec. per. ☿ ÷ ⊙r)$^{\frac{4}{3}}$ = 2.420721 = log. 1.226 ⊕r.vec.; (sec. per. ♂ ÷ ⊕)$^{\frac{3}{4}}$ = 1.225 ....(34.)

The atmospheric limit at the central locus of infra-asteroidal rupture, is at Jupiter's mean aphelion. For log. (sec. per. ⊕ ÷ ⊙r)$^{\frac{4}{3}}$ = 3.068927; log. (mean aph. ♃ ÷ ⊙r) = 3.066743.............................(35.)

The atmospheric limit at the rupturing locus of Mars, is near the rupturing limit of Saturn. For log. (sec. per. ♂ ÷ ⊙r)$^{\frac{4}{3}}$ ÷ ⊕r.vec. = .934212 ; log. sec. per. ♄ = .941236 ; .941236 — .934212 = .007024 = log. 1.0163. This indicates a similarity of contraction at the centre (6) and at the outer limit of the belt..(36.)

The atmospheric limits of the Venus belt, as determined by reference to the rupturing position of Mercury, are in or near the Earth belt. For log. (♀ ÷ sec. per. ☿)$^{\frac{4}{3}}$ ÷ ⊕r. vec. = $\bar{1}$.942238 @ .024175 ; log. ⊕ = $\bar{1}$.969540 @ .028463...(37.)

The atmospheric limits of the Earth belt, referred to the rupturing position of Mercury, are within the Mars belt. For log. (⊕ ÷ sec. per. ☿)$^{\frac{4}{3}}$ = .131591 @ .210155 ; log. ♂ = .117620 @ .239631.................(38.)

The atmospheric limits of the Mars belt, referred to the rupturing position of Mercury, are within the asteroidal belt......................(39.)

The atmospheric limit at Venus's mean aphelion, referred to Mercury's mean locus, is at Earth's rupturing locus. For log. (mean aph. ♀ ÷ ☿)$^{\frac{4}{3}}$ = .382120 ; log. (sec. per. ⊕ ÷ ☿) = .381719......................(40.)

The atmospheric limit at Earth's initial locus, referred to the initial locus of Mercury, is at the mean perihelion of Mars. For log. (sec. aph. ⊕ ÷ sec. aph. ☿)$^{\frac{4}{3}}$ = .466876 ; log. (mean per ♂ ÷ sec. aph. ☿) = .468819...(41.)

The initial locus of Earth is at the mean aphelion thermal radius of Venus. For log. 1.4232 (mean aph. ♀ ÷ ⊕) = .027677 ; log sec. aph. ⊕ = .028463...(42.)

The atmospheric limit at the rupturing locus of Mars (36), referred to the rupturing position of Venus, is near the mean aphelion of Mars. For log. (sec. per. ♂ ÷ sec. per. ♀)$^{\frac{4}{3}}$ = .214318 ; log. mean aph. ♂ = .215944...(43.)

The inner atmospheric limit of the Jupiter belt, referred to the rupturing position of Venus, is at Saturn's mean distance. For log. (sec. per. ♃ ÷ sec. per. ♀)$^{\frac{4}{3}}$ = .976134 ; log. ♄ = .979496..................(44.)

The outer atmospheric or initial limit of the Jupiter belt, referred to the rupturing position of Venus, is near Saturn's initial locus. For log. (sec. aph. ♃ ÷ sec. per. ♀)$^{\frac{4}{3}}$ = 1.046666 ; log. sec. aph. ♄ = 1.014657 ; 1.046666 ÷ 1.014657 = .032009 = log. 1.077 = log. (sec. aph. ÷ mean ♅). See (3) to (5), (20)...(45.)

The inner atmospheric limit of the Saturn belt, referred to the rupturing limit of Venus, is at the secular aphelion of Uranus. For log. (sec. per ♄ ÷ sec. per. ♀)$^{\frac{4}{3}}$ = 1.312473 ; log. sec. aph. ♅ = 1.315531........(46.)

The outer atmospheric limit of the Saturn belt, referred to the rupturing position of Mars, is also at the secular aphelion of Uranus. For log. (sec. aph. ♄ ÷ sec. per. ♂)$^{\frac{4}{3}}$ = 1.313669..............................(47.)

The inner atmospheric limit of the Jupiter belt, referred to Earth's rupturing position, is near Saturn's rupturing position. For log. (sec. per. ♃ ÷ sec. per. ⊕)$^{\frac{4}{3}}$ = .928796 ; log. sec. per ♄ = .941236 ; .941236 — .928796 = log 1.029. .... (48.)

The outer atmospheric limit of the Jupiter belt, referred to Earth's rupturing position, is at the mean aphelion of Saturn. For log. (sec. aph. ♃ ÷ sec. per. ⊕)$^{\frac{4}{2}}$ = .999328 ; log. mean aph. ♄ = 1.000003..........(49.)

The mean atmospheric limit of the Saturn belt, referred to Earth's mean position, is near the mean aphelion of Uranus. For log. (♄ ÷ ⊕)$^{\frac{4}{3}}$= 1.305995 ; log. mean aph. ♅ = 1.301989...........................(50.)

The atmospheric limit at Jupiter's mean aphelion, referred to the rupturing position of Mars, is at Saturn's rupturing position. For log. (mean aph. ♃ ÷ sec. per. ♂)$^{\frac{4}{3}}$ = .940244 ; log. sec. per ♄ = .941236......(51.)

The mean atmospheric limit of the Uranus belt, referred to Jupiter's rupturing position, is at Neptune's mean aphelion. For log. (♅ ÷ sec. per. ♃)$^{\frac{4}{3}}$ = 1.480913 ; log. mean aph. ♆ = 1.481951.....................(52.)

The same limit (52), referred to Jupiter's mean perihelion, is at Neptune's mean locus. For log. (♅ ÷ mean per. ♃)$^{\frac{4}{3}}$ = 1.478215 ; log. ♆ = 1.477611..(53.)

The same limit, referred to Jupiter's mean position, is at Neptune's rupturing position. For log. (♅ ÷ ♃)$^{\frac{4}{3}}$ = 1.471828 ; log. sec. per. ♆ = 1.471268..(54.)

The important influence of Earth's position at a centre of early nucleal condensation, is also shown by the simplicity of relations between Earth's radius vector and the secular epicyclical undulations of the supra-asteroidal planets.

Earth and Sun are convertible points of suspension, for a linear pendulum equivalent to the secular excursion of Uranus. For 3 ÷ 38.365 = .0782 ; the maximum eccentricity of Uranus is .0789 ; see (20) (21).........(55.)

The excursion of Saturn is nearly equivalent to the atmospheric limit of a nucleus which has Earth's thermal radius ($1.4232^{\frac{4}{3}}$ = 1.601). For 1.601 ÷ 19.078 = .0839 ; the maxium eccentricity of Saturn is .0843.........(56.)

The excursion of Jupiter is equivalent to the mean radius of rotating inertia at Earth's mean distance ($\sqrt{.4}$ = .6325). For .6325 ÷ 10.406 = .06078 ; the maximum eccentricity of Jupiter is .06083...............(57.)

The excursion of Neptune is in the inverse ratio of its own coefficient

($\frac{8}{7}$) and in the direct ratio of the coefficient of Uranus ($\frac{7}{8}$), in the abscissas of the stellar-solar paraboloid which has its vertex at the point of complete solar aggregation.* For $\frac{7}{8} \div 60.074 = .0147$; the maximum eccentricity of Neptune is .0145. ....(58.)

The following table shows the closeness of approximation (Theoretical less Observed $\div$ Observed), in each of the foregoing comparisons :

1	—.0039	15	.0052†	30	—.0010	45	—.0009
2	.0197	16	—.0028†	31	.0001	46	—.0070
3	—.0158	17	.0001	32	—.0005	47	—.0043
4	.0109	18	—.0006	33	.0002	48	.0291
5	—.0014	19	.0019	34	.0008	49	—.0016
6	.0001	20	.0006	35	.0050	50	.0093
7	.0033	21	—.0224	36	.0006	51	—.0023
8	.0074	22	—.0013	37	.0000	52	—.0025
9	.0000	23	—.0012	38	.0000	53	—.0014
10	.0070†	24	.0010	39	.0000	54	—.0013
11	—.0023†	25	.0010	40	.0009	55	.0030
12	.0169†	26	.0001	41	—.0045	56	—.0050
13	.0238†	27	.0011	42	—.0018	57	—.0008
14	.0319†	28	.0039	43	—.0038	58	.0014
		29	.0019	44	—.0078		

One of the most important corollaries of the theory of universal gravitation, is tersely stated by Stockwell,‡ as follows : "The amount by which the elements of any planet may ultimately deviate from their mean values can only be determined by the simultaneous integration of the differential equations of these elements, which is equivalent to the summation of all the infinitesimal variations arising from the disturbing force of all the planets of the system during the lapse of an infinite period of time." Therefore, within the limits of secular eccentricity, the result is the same *as if* the nebular hypothesis were true.

There should, then, be tendencies, in the neighborhood of every inert particle which floats in an elastic medium, to the formation of harmonic nodes of various kinds, and the sum of such tendencies should fix loci of cosmical aggregation before there had been any considerable shapings of definite mass. The subsequent values of relative mass would depend upon mutual conditions of equilibrium between various forms of living force.

But such accordances as the foregoing, however interesting, and however striking they might be deemed, would furnish no more conclusive evidence of the nebular theory, as popularly interpreted, than of the Cartesian vortices. All assumptions as to the nature of ultimate physical force, are now, and perhaps always will be, mere assumptions ; still, like geometrical diagrams, they may help to fix the mind upon ultimate physical resultants, and thus serve a useful purpose.

* Ib. † According to Herschel's estimate.

‡ Smithsonian Contributions, 232. viii.

Stockwell also remarks* that "a comparison of the values which the different solutions give for the superior limit of the eccentricity of the Earth's orbit, has suggested the inquiry whether there may not be some unknown physical relation between the masses and mean distances of the different planets." If such a relation exists, the constancy of the mean distances would also seem to imply a like constancy of absolute or relative masses, and the inquiry naturally arises, which is the logical antecedent; whether distance has determined mass or mass has determined distance.

There is abundant evidence on the one hand, of "cosmical dust," and meteorites, which are contributing to the enlargement of the sun and the planets; on the other, of internal convulsions, which are occasionally ejecting materials beyond the reach of primitive attraction. It is commonly believed that the enlarging tendencies predominate, and that the sum of all physical tendencies is towards stagnation, death, and universal gloom. Such a belief seems to me erroneous, and based upon limited considerations. It is not easy, as yet, to trace all the compensative and restorative energies, but some of them are strongly indicated by the various cosmical relations which, through all periodic and secular inequalities, tend to maintain the stability of planetary orbits.

In judging of the probable logical antecedence, it is well to remember, that all of the correlations which I have pointed out have been based upon general considerations of oscillatory centres, as influenced by radial, tangential, superficial, and volumetric disturbances, with reference to simple centres of inertia, and entirely independent of mass. My own convictions that spirit must necessarily, both logically and chronologically, take precedence of matter, have been strengthened by my investigations. Others, who have been accustomed to look more exclusively to physical influences, may perhaps be differently impressed by them. If they can give any physical explanation of the instantaneous action of gravity at all distances, or if they can frame any satisfactory hypothesis to account for such action except by constant spiritual activity, it will be gladly welcomed by all sincere seekers after truth. If, on the other hand, they admit that instantaneous action is incompatible with inertia, they may find that their own studies of nature lead them to a sure recognition of the supernatural, as an essential element of sound and catholic philosophy.

If a nebulous mass were to be divided by some internal convulsion, the ruptured portions would be projected from their common centre of gravity to distances varying inversely as their masses. If the first rupture were simple and one mass were much larger than the other, it could not obey the tendency to revolve about the common centre of gravity in a time proportional to the $\frac{3}{2}$ power of the distance, but the tendency might be manifested in other equivalent ways. The synchronism of linear oscillations through twice the diameter, and orbital oscillations through the circumference of a circle, points to a possible mode of such manifestation, by the removal of the larger mass to such distance as would allow the linear oscillations, or equivalent

* Op. cit., xvii.

tangential oscillations. This is the case with Sun and Jupiter. For if we represent the mass of Jupiter by 1, and Sun's mass by n, the secular perihelion distance of their centres, $.9391726 \times 5.2028 \times 214.86\,r = (n + 2)\,r$; and $n = 1047.876$. Bessel's value is 1047.879 .... (59.)

The atmospheric relation of Saturn to Jupiter (51) is further shown by the equality of nebular *vires vivæ*. For orbital $v.\ v. \propto \frac{m}{r}$; the nebulous mass at Jupiter's thermal radius vector,* is 1.4232^3, and the orbital $v.\ v.$ at the corresponding atmospheric limit is $1.4232^3 \div 1.4232^{\frac{4}{3}} = 1.4232^{\frac{5}{3}} = 1.8007$, which is Alexander's ratio.† The $v.\ v.$ of constrained rotation varies as the square of the orbital $v.\ v.$, or, in the present case, as 3.2426 to 1, which would require masses in the ratio of 1 to 3.2426 to give equality of $v.\ v.$; provided the primitive orbits were circular. We have seen, however, (6), that the infra asteroidal centre is at $1.01688 \times$ Earth's mean distance, and the Jovi-Telluric connections (3) suggest the probability of a similar eccentricity in the primitive Jovi-Saturnian belt. $(1.4232^{\frac{5}{3}} \times 1.01688)^2 = 3.3525$; $1047.876 \times 3.3525 = 3513.47$. Bessel's ratio of Sun to Saturn is 3501.6; Le Verrier's 3512 .... (60.)

Saturn's position at the nebular centre of planetary inertia would be likely to establish permanent records of equality in still other forms of *vis viva*. We accordingly find that the ratio of Neptune's to Saturn's mass seems to be due to nucleal *vis viva* (4) when they were both at atmospheric limits. For the $v.\ v.$ of nucleal rotation varies as $\left(\frac{1}{d}\right)^{\frac{3}{2}}$; the $\frac{3}{2}$ power of Neptune's, divided by Saturn's mean distance, $= 5.587$; $5.587 \times 3513.47 = 19630$. Newcomb's ratio of Sun to Neptune, as deduced from the perturbations of Uranus is 19700 .... (61.)

The mass ratio of Uranus to Saturn seems to be due to atmospheric *vis viva* when their nucleal condensation began. For the $v.\ v.$ of rotation at the atmospheric limit varies as $\left(\frac{1}{d}\right)^{\frac{8}{3}}$; the $\frac{8}{3}$ power of mean distance (♅ ÷ ♄) is 6.444; $6.444 \times 3513.47 = 22641$; Newcomb's ratio of Sun to Uranus is 22600 ± 100 .... (62.)

The mass ratio of Saturn to Earth seems to be due to equality of rotating *vis viva*. For the $v.\ v.$ of rotation in a contracting nucleus varies as $\left(\frac{1}{d}\right)^2$; the square of mean distance (♄ ÷ ⊕) is 90.99; $90.99 \times 3513.47 = 319690$. The ratio of Sun to Earth is still a mooted question...... (63.)

These theoretical masses are such as to contribute to the stability of the system, by giving equality between various forms of opposing $v.\ v.$ at culminating points of opposing disturbance.

With the Sun expanded to Neptune's mean aphelion and rotating as a

* Fundamental Propositions, III, IV.

† Proc. S. P. A. xii. 394.

nebulous mass, at the beginning of interplanetary condensation (secular aphelion) the mean *vires vivæ* of the outer two-planet belts are equal. For the internucleal $v.\ v. \propto md^2$; log. md^2 (♆ × ♅) = 5.656948;* log. md^2 (♄ × ♃) = 5.656817. See also (58)........................(64.)

With Neptune at secular aphelion the mean *vires vivæ* of the outer and inner limits of the outer two-planet belts are equal. For under uniform æthereal resistance the *v. v.* is proportioned to the product of the mass by the trajectory, and the mean orbital trajectory is proportioned to the mean distance. Taking Uranus, Saturn and Jupiter at their mean distances, log md (♆ × ♄) = 3.334505; log md (♅ × ♄) = 3.333751..(65.)

With Jupiter at Sun's nucleal surface, and the outer planets at tidal crests (secular aphelion), the mean *v. v.* of the two outer = mean *v. v.* of the two inner planets. For the *v. v.* of rotation in a shrinking nucleus $\propto m \div d^2$; the orbital $v.\ v. \propto m \div d$; log. (♃ *rot.* × ♆ *orb.*) *v. v.* = $\bar{2}.480226$; log. (♅ × ♄) *orb. v. v.* = $\bar{2}.478969$.. ....(66.)

In my equation of figurate powers, log. (♆ × ♅^{3} × ♃^{6}) = 8.069488; log. ♄^{10} = 8.091570; log. ♄ theoretical mass = .806949 ....(67.)

The internucleal *v. v.* ($\propto md^2$) of Saturn is equivalent to the mean internucleal *v. v.* of the supra-asteroidal belt. For if we consider Neptune at secular aphelion, Uranus and Saturn at mean distance, and Jupiter at secular perihelion, log. md^2 for ♆ = 3.029720; for ♅ = 2.565859; for ♄ = 2.768149; for ♃ = 2.712548; (3.029720 + 2.565859 + 2.768149 + 2.712548) ÷ 4 = 2.769069 ....(68.)

The mean *v. v.* of æthereal rupturing projection (md) in the supra-asteroidal belt is equivalent to that of the Sun (59). For log. [mass (♆ × ♅ × ♄ × ♃) $\frac{1}{4}$ ÷ ☉ × secular perihelion ♆ ÷ ☉ radius] = $\frac{1}{4}$ ($\bar{5}.707091 + \bar{5}.645107 + \bar{4}.454264 + \bar{4}.979691$) + 3.803423 = $\bar{1}.999961$; log. mass ☉ ÷ ☉ radius = .000000..................................(69.)

Jupiter's mass is nearly equivalent to the mean mass of Sun, Earth and Saturn. For log. $\frac{1}{3}$ (☉ × ⊕ × ♄) = 1.338072; log. ♃ = 1.334584...(70.)

II. *Chemical Atoms, Molecules and Volumes.*

In accordance with a suggestion of Professor Robert E. Rogers, I have endeavored to find what modes of central force will best represent some of the most general forms of chemical activity, more especially those which are the basis of the law of Avogadro and Ampére, of combination by volume, and of approximate constancy in the product of atomic weight by specific heat.

The simplicity of the ratio, between the energy of H_2O and the solar energy at Earth's mean distance,† furnishes good grounds for such an investigation, while the record of a parabolic orbit, connecting the Sun with the nearest fixed stars,‡ indicates a proper course for conducting it. Al-

* With Uranus as unit of mass, and Earth as unit of distance.

† *Ante* xii, 394; xiii, 142.

‡ *Ante*, xii, 523, and subsequent papers.

though there may be some doubt as to the degree of certainty which belongs to the recent hypotheses of internal gaseous structure, there can be none as to the graphic representation of orbital activities under forces varying inversely as the square of the distance. Circular orbits denote constancy of relations between radial and tangential forces; elliptic orbits, variability of relations accompanied by cyclical oscillations; parabolic orbits, variability of relations without cyclical oscillation; hyperbolic orbits, variability of relations complicated by the action of extraneous force.

In a rotating mass, the orbits of the several particles are circular. If the uniform velocity of any particle in the equatorial plane is less than $\sqrt{fr}$, the mean action of the central force is impeded by internal collisions or resistances. If the velocities of all the particles in the plane vary precisely as $\sqrt{fr}$, there is a condition of perfect fluidity, marking a limit between complete aggregation and incipient dissociation. Any cyclic variations of velocity between constant limits indicate elliptic orbits, with tendencies to aggregation through collisions near the perifocal apse. A perifocal velocity of $\sqrt{2fr}$ marks a parabolic orbit, and a limit between complete dissociation and incipient association. A velocity greater than $\sqrt{2fr}$ is hyperbolic, indicating the intervention of a third force in addition to the mutual action between the two principal centres of reference.

If all physical forces are propagated by æthereal undulations between resisting points, those points tend naturally to nodal, and from internodal positions. In order to maintain uniformity in the wave velocity, the æthereal molecules must be uniform, not only in volume, but also in aggregate inertia. As the inertia of the resisting points increases, the inertia due to internal æthereal motions, should, therefore, diminish, and *vice versâ*. In other words, the uniform elementary volume may be represented by the product of atomic weight by specific heat, and the laws of Boyle (or Mariotte), Charles, and Avogadro, follow as simple and necessary corollaries.

In order that uniform undulations should produce motion, there must be at least two points of resistance. Those points would approach each other until the interior undulating resistance equaled the exterior undulatory pressures, when their motion would be converted into rotation or into orbital revolution. Their common centre of revolution might become the centre of a new elementary volume, thus giving rise to the various laws of combination by volume, combination without condensation, condensation of two volumes into one, three volumes into two, or four volumes into two, as well as to general artiad and perissad quantivalence.

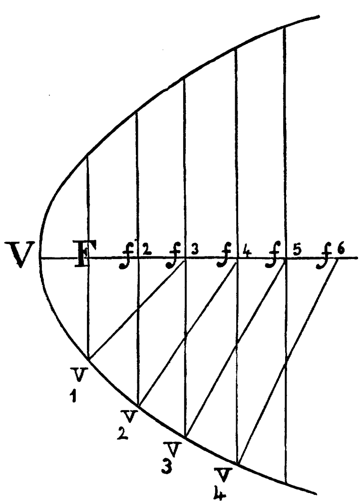
When perifocal collisions change parabolic or elliptic into circular orbits, there should be increasing density towards the principal centre of the system. Further collisions and condensations would produce tendencies to both nucleal and atmospheric* aggregations, and consequent binary groupings. These laws are exemplified in the solar system, by the general division into an intra asteroidal and an extra-asteroidal belt, and by the subdivis-

* *Ante*, xiv, 622, sqq.

ion of each belt into two pairs, the inner belt being denser than the outer, and the inner member of each pair being denser than its companion ; Mercury being denser than Venus ; Earth, than Mars ; Jupiter, than Saturn ; Uranus, than Neptune. This arrangement towards the Sun as a principal centre, appears, however, to be of more recent date than the tendency to condensation in the Telluric belt, for Earth is denser than Venus, and the great secular ellipticities of Mars and Mercury suggest the likelihood of a quasi-cometary origin. Similar tendencies would contribute to the chemical grouping of atoms by pairs, which is essential for polarity and for the already-enumerated laws of chemical combination.

In the "nascent state," particles may be regarded either as parabolically perifocal, with the velocity of complete dissociation from a given centre, or as relatively at rest, and ready to obey the slightest impulses of central force. The mean *vis viva* of a system formed by two such particles would be $m \times (\sqrt{2})^2 + m \times 0 = 2\,m \times 1$, representing a change from parabolic to circular orbits and a condensation of two volumes into one.

At the parabolic limit between complete dissociation and incipient aggregation, if the focal abscissa $x_0 = VF$, is taken as the unit of wave-length, the value of the successive ordinates, as well as the velocity communicated by uniform wave influence acting through the entire length of the ordinates, will be represented by $\sqrt{4\,x_n}$; the resulting *vis viva*, and the consequent length of path, or major axis, communicable against uniform resistance, by $4\,x_n$; the successive differences of major axis, by 4. Each normal, $v_n\, f_{n+2}$, equals the next ordinate, $v_{n+1} f_{n+1}$; there are, therefore, triple tendencies, both in the axis of abscissas, and on each branch of the curve, to successive differences of 4 in the major axes of aggregation, in consequence of the meeting of abscissal, ordinal, and normal waves in the axis, and the meeting of tangential, normal, and abscissal waves upon the curve. At each node of aggregating collision two of the wave systems are due to normally alternating rectangular oscillations,* the third serving as a link between the axial and the peripheral waves. The bisection of the normals, by their equivalent ordinates, adds importance to the normal major axes, and increases the tendency to aggregation at their respective centres of gravity.



* "Fundamental Propositions," 13.

Chemical molecules and atoms are so small that we are unable, at present, to show, so conclusively as in cosmical gravitation, that the "nascent" velocity, or the mean radial velocity at the limit between complete dissociation and incipient aggregation, is equivalent to the velocity of light. But the analogies, which are here presented, are strengthened by the frequent vivid, luminous and thermal accompaniments of chemical change, and by the electric polarity of combining elements. It seems, therefore, reasonably certain that the same limiting unity of velocity and *vis viva*, which can be easily traced in light, heat, electricity and gravitation, is also fundamentally efficient in chemical affinity. M. Aymonnet, in his communication of a "nouvelle méthode pour étudier les spectres calorifiques,"* says: "Je ferai remarquer, avant de terminer, que l'étude des spectres calorifiques d'absorption, faite avec des corps portés à diverses températures, peut et doit conduire à la connaissance de lois physiques reliant les phénomènes d'association et de dissociation des corps aux phénomènes calorifiques et lumineux." In another paper recently presented to the French Academy, "sur le rapport des deux chaleurs spécifiques d'un gaz,"† M. Ch. Simon deduces the theoretical ratio C : c : : 1.4 : 1. The first attempt at a solution of the problem upon *a priori* grounds, appears to have been Professor Newcomb's,‡ who found from the hypothesis of actual collisions, the ratio 5 : 3 if the particles were hard and spherical, or 4 : 3 if they were hard and not spherical; the second, my own,§ based on the general consideration of all internal motions, which led to the ratio 1.4232 : 1; the third, M. Simon's, which took account of rotations and neglected other internal vibrations.

* Comptes Rendus, lxxxiii, 1102—4, Dec. 4, 1876.
† Ib. 727, Oct. 16, 1876. ‡ Proc. A. A. S., v., 112. § *Ante*, xiv, 651.